MTH 203: Introduction to Groups and Symmetry

Homework VII

(Due 02/11/2022)

Problems for submission

1. Show that A_4 has a unique subgroup H of order 4, which is normal and isomorphic to the Klein 4-group. Also, determine the group A_4/H.

2. Show that for a prime p, an element in S_n has order p if and only if its cycle decomposition is a product of commuting p-cycles. Provide a counterexample that shows that this assertion does not hold when p is composite.

3. Show that for $n \geq 3$, A_n is generated by 3-cycles.

Problems for practice

1. Show that for $n \geq 3$, every element in A_n is a product of n-cycles.

2. What is the the largest possible order of an element in S_n? (This is called the Landau number.)

3. Show that a group of order 6 is either isomorphic to C_6 or S_3.